- Stat. complexity
- Algorithmic complexity

Having a sequence s

Frequency of subsequences s_n

$$\nu(s_n) = \lim_{n \to \infty} \frac{N(s_n)}{N}$$

For UTM:

$$s_n \sim N$$

Lyapunov analysis

$$\vec{x}(0) + \vec{w}(0) \qquad |\vec{w}(0)| < |\vec{x}(0)|$$

$$\frac{d\vec{w}(t)}{dt} = \hat{J}\left(\vec{x}(0), \vec{x}(t)\right)\vec{w}(t)$$

Oseledec multiplicative theorem

$$\lim_{t \to \infty} \frac{1}{t} \left| \hat{J}^{+}(t) J(t) \right|^{\frac{1}{2}} = \lambda_n$$

$$w(t) = e^{\lambda_n t}$$

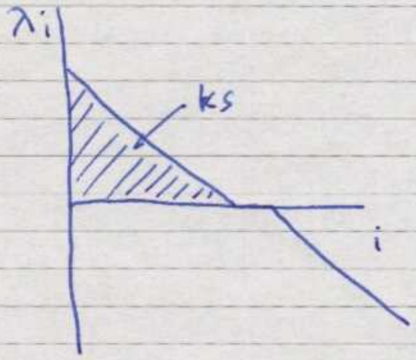
KS entropy:

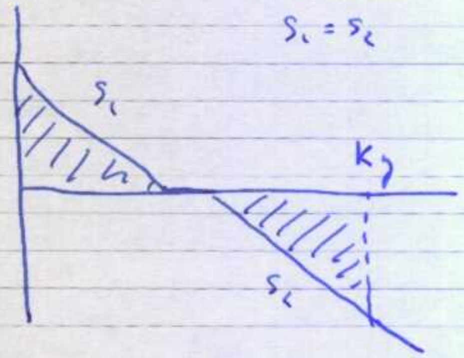
$$h_{KS} = \sum_i \lambda_i$$

The other Lyapunov exponents

$$\lambda_2 = (\vec{w}_1 \wedge \vec{w}_2)\, \vec{w}_{12}$$

$$J(\vec{w}_{12}) = \lambda_1 + \lambda_2 - \lambda_1 = \lambda_2$$



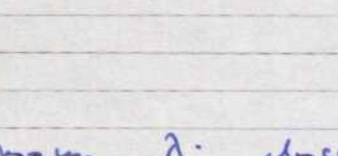


important

if you have many λ_i close to $0 \rightarrow$ for those coordinates the system can stay for long in the same condition.

They are difficult to estimate:
having initially an orthogonal set it quickly collapses to one dimension (maximal λ_i)
if you renormalize them continuously, they converge to true λ.

The trick:



inverte G_i and run back. The minimal λ_i becomes the maximal.

Other forms of unpredictable evolution:
cellular automata + chaotic

0,011000 : time of period $T \sim e^L$

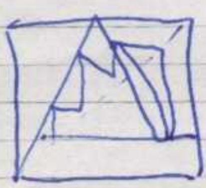
changed

0 1 1 0 0 0 0 1
0 1 1 1 0 0 1

λ_{max} ← perturbation spreads

↑ analogous to Lyapunov exponents

The case when $\lambda_{max} < 0$



← periodic

$$x^{t+1} = (1-\varepsilon) f(x^t) + \frac{\varepsilon}{2}\left[f(x^t_{i-1}) + f(x^t_{i+1}) \right]$$

$$0 < \varepsilon < 1$$

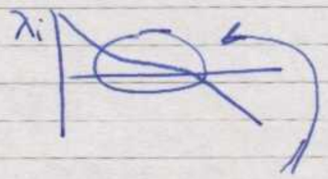
$$\lambda_{max}(\varepsilon) < 0$$

similar to CA (losing)

0 ? 1 ε

transition
from periodic to unpredictable

If there are many slowly varying variables:



fluids → hydrodynamic modes ← described by the Lyapunov eigenvectors

You cannot separate fast and slow vars. for the case of related hierarchies

similar → folding